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## Bridging FEM and Artificial Neural Network in gating system design for smart 3D sand casting

Ahmed KTARI<sup>a\*</sup>, Mohamed ELMANSORI<sup>a,b</sup>

<sup>a</sup>MSMP-EA7350, Arts et Métiers ParisTech, 2 cours des Arts et Métiers, 13617 Aix-en-Provence, France

<sup>b</sup>Department of Mechanical Engineering, Texas A&M University, College Station, TX, 77840, USA

\* Corresponding author. E-mail address: [ahmed.ktari@ensam.eu](mailto:ahmed.ktari@ensam.eu)

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### Abstract

A relatively new methodology bridging FEM and Artificial Neural Network (ANN) is proposed and validated in this study to optimize the gating system design for smart 3D sand casting. This methodology was applied on the case of 3D sand casting of a simple plate with aluminum alloy (EN AC-44200). Several mold-filling simulations are performed with the commercial FE code ProCast® by using a combination of the studied gating system design parameters, selected from Taguchi orthogonal array. Signal to noise (S/N) ratio and analysis of variance (ANOVA) are then employed to analyze the contributions of the studied design parameters on the molten metal velocity at the ingate. The significant parameters and their corresponding FE simulations are used to train and validate the ANN model. It is found that ANN simulator can rapidly predict the ingate velocity for any combination of the significant gating system design parameters covering the studied design space.

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**Keywords:** 3D sand casting; Gating system design; FEM simulation; Artificial Neural Network; Ingate velocity.

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### 1. Introduction

The design of molds gating system is considered as a key point to which we need pay special attention, since it permit to control the melt flow behavior in the mold cavity. The transition from laminar to turbulent flow during the mold filling can lead to drastic effects on the quality of castings [1].

The filling process is typically comprised of free surface flow of the metal front inside the mold cavity. The exposure of liquid metals to air and moisture during free surface flows leads to the formation of surface oxide films [2]. Folding of free dry oxide surfaces result in harmful double oxide films named bifilms [3,4]. Because these bifilms are necessarily folded on their dry sides during their creation, they act as cracks and initiate failure [5]. The rate of bifilm formation increases with increase in turbulence as the oxide layers continuously stretch, rupture and regrow [6,7].

Entrained defects as oxide inclusions, sand inclusions, blowholes, bubbles, bifilms among others are significantly detrimental to the castings mechanical properties [1, 8-10]. Hence, oxide films can affects tensile strength [11-13], fracture strength [14], fatigue life [15] and also act as initiation sites for shrinkage [16] and hydrogen gas pore formation [17] in castings.

As a result, to reduce defects due to entrainment since they cause 80% of the total effective problems in castings [18], it is critical to design proper gating systems that permit to control the liquid metal flow during the mold filling phase by reducing the velocity at ingate to less than 0.5 m/s [19-21]. This will improve product quality and foundry productivity.

In the past, several researchers have experimented with various parameters to design straight sprue based on the basic principles of fluid dynamics. Most of the current design knowledge on gating systems are derived from trial and error



The cast part and gating system geometry were adopted from Yang et al. (1998) [34]. The advantage of the undercut at the base of pouring basin include minimizing the occurrence of splashing and the sprue cover prevents metal from directly falling into the sprue.

Given that Taguchi method requires a low number of experiment combinations, it can be applied to obtain useful information for reducing the number of experiments and converting quality characteristics into a signal to noise (S/N) ratio. Indeed, Taguchi orthogonal array is applied in this study to facilitate experimental design process as shown in Table 1.

Table 1. Gating system parameters and levels used in experimental plan following Taguchi's standard L27 (311) orthogonal array

Training case N°	A	B	C	D	E	F	G	H	I	J	K
1	12	12	6	160	90	30	8	30	10	40	24
2	12	12	6	160	110	50	9	40	15	50	32
3	12	12	6	160	130	70	10	50	20	60	40
4	12	15	8	200	90	30	8	40	15	50	40
5	12	15	8	200	110	50	9	50	20	60	24
6	12	15	8	200	130	70	10	30	10	40	32
7	12	18	10	240	90	30	8	50	20	60	32
8	12	18	10	240	110	50	9	30	10	40	40
9	12	18	10	240	130	70	10	40	15	50	24
10	16	12	8	240	90	50	10	30	15	60	24
11	16	12	8	240	110	70	8	40	20	40	32
12	16	12	8	240	130	30	9	50	10	50	40
13	16	15	10	160	90	50	10	40	20	40	40
14	16	15	10	160	110	70	8	50	10	50	24
15	16	15	10	160	130	30	9	30	15	60	32
16	16	18	6	200	90	50	10	50	10	50	32
17	16	18	6	200	110	70	8	30	15	60	40
18	16	18	6	200	130	30	9	40	20	40	24
19	16	12	10	200	90	70	9	30	20	50	24
20	20	12	10	200	110	30	10	40	10	60	32
21	20	12	10	200	130	50	8	50	15	40	40
22	20	15	6	240	90	70	9	40	10	60	40
23	20	15	6	240	110	30	10	50	15	40	24
24	20	15	6	240	130	50	8	30	20	50	32
25	20	18	8	160	90	70	9	50	15	40	32
26	20	18	8	160	110	30	10	30	20	50	40
27	20	18	8	160	130	50	8	40	10	60	24

#### 4. 3D-FE modelling and computation

##### 4.1. Casting process parameters modelling

A thermo-hydraulic fluid flow modelling of the mold filling studied cases, applying thermally coupled Navier-Stokes equations, was performed via ProCAST® FE software with identical inlet fluid and boundary conditions. The coupled 3D model was chosen to make numerical model as close as possible to experimental conditions and to ensure that all selected combinations of gating system design parameters allows a complete filling of the mold cavity (i.e. avoid misrun risk). The modelling of the casting cases require geometrical information of the CAD model including the part, the gating system, and the sand mold. Hence, all studied cases of solid CAD models are generated using CATIA® V5 software before being imported in ProCAST®. In this modeling, aluminum alloy EN AC-44200 is used for the cast part.

These alloys have a wide range of applications in automotive and aerospace industries due to their excellent castability, mechanical properties as well as good corrosion and wear resistance. The model is meshed with more than  $2.36 \cdot 10^6$  tetrahedral elements with an approximate dimension of 1 mm for the part and the gating system and 3 mm for the mold

(Figure 3). Temperature dependent thermo-physical material properties for thermal conductivity, specific heat capacity and density are used. An initial temperature of 750 °C is applied to the inlet, which represents the pouring temperature of the molten metal. An inlet metal flow rate of 225 cm³/s into the pouring basin was maintained constant during the filling phase. The casting is cooled to room temperature in air with a convection coefficient of 10 W.m⁻².°C⁻¹ applied to all six mold surfaces. The room (sink) temperature of the surrounding air is 25°C. In addition, a heat transfer coefficient of 350 W.m⁻².°C⁻¹ is applied at the metal/sand mold interface [35].

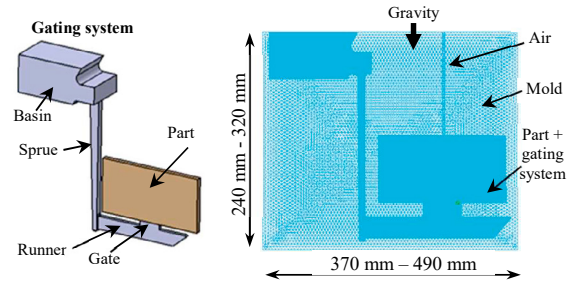


Fig. 3. CAD and FE mesh applied on the studied model

##### 4.2. Gating velocity and S/N ratio's calculation

The evolution of the ingate velocity (i.e. velocity measured on the top of the gate) during the filling phase is monitored by using probes, which placed at the end of the ingate along the casting mid-plane. The maximum value of the ingate velocity is selected for all simulated cases and then signal to noise ratios are calculated according to Taguchi method. This method is considered as a quantitative analysis, largely used to identify control parameters that affect mean and variation of the quality characteristics. A high value of S/N ratio indicates the optimum quality with minimum variation [36]. The S/N ratio, expressed in dB units, can be defined as Equation 1 by logarithmic function based on the Mean Square Deviation around target for the output characteristic.

$$S/N = -10 \log \left( \frac{1}{n} \sum_{i=1}^n Y_i^2 \right) \quad (1)$$

Where n is a total number of tests in a trial and  $Y_i$  represents the value of the measured ingate velocity.

In this study, the lower the better quality characteristic for the ingate velocity can be taken for obtaining the optimal casting quality. This parameter is chosen due to its importance in the casting quality. In fact, decreasing of the ingate velocity can reduce turbulences and entrainment, which can lead to several casting defects [1].

The mold filling simulation, for two different filling system design parameters combination, shows different flow behavior of the melt metal inside the mold cavity (Figure 4). A laminar flow was observed (Figure 4(a)) during the filling process of the simulation N°3 ( $V_{\text{ingate}} = 0.416$  m/s) which entirely adheres to the critical ingate velocity condition described above. However, when this velocity exceed to the critical value as shown in Figure 4(b) ( $V_{\text{ingate}} = 1.09$  m/s), the melt metal form a fountain in the gating zone followed by severe undesired melt

turbulences which lead to the formation of several defects in the final casted part.

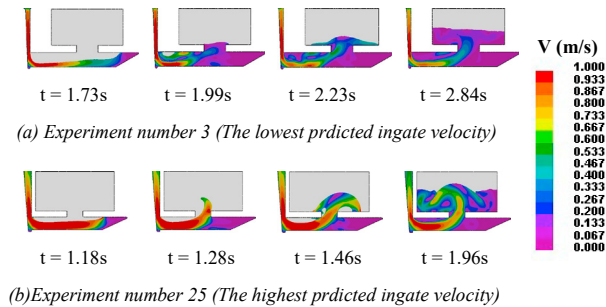


Fig. 4. Melt flow velocity distribution during the sand mold filling.

### 5. Statistical significance of the studied gating system parameters

Based on the simulation results of the selected training cases, the response of each factor to its individual level was calculated by averaging the S/N ratios of all experiments at each level and factor. After that, analyze of variance (ANOVA) test was performed to identify the most effective gating system parameters that significantly affect the filling ingate velocity. The ANOVA was established on the sum of the square (SS), the degree of freedom (DOF), the variance (V) and the percentage of the contribution to the total variation [37]. As shown in table 2 at its confidence level of 90% ( $F_{0.1,2,26}=2.519$ ), four design parameters (A, B, C, D and J) were found as significant for the ingate velocity. However, I is also considered as significant parameters given that its F-ratio value is close to the acceptance value. In summary, the ANOVA result shows that gating dimensions A, B, C, D, I and J play a crucial role on the value of the molten metal ingate velocity during the mold filling.

Table 2. ANOVA results for signal to noise for the ingate velocity response

Gating parameters	Sum of square	DOF	Variance	F-ratio	Contribution (%)	Rank	Significant (confidence >90%)
A	12.032	2	6.016	4.094	14.155	2	YES
B	20.506	2	10.253	6.977	24.125	1	YES
C	10.578	2	5.289	3.599	12.445	4	YES
D	8.553	2	4.276	2.910	10.062	5	YES
E	1.266	2	0.633	0.431	1.489	9	NO
F	0.840	2	0.420	0.286	0.988	11	NO
G	4.076	2	2.038	1.387	4.796	7	NO
H	0.977	2	0.489	0.332	1.150	10	NO
I	6.983	2	3.492	2.376	8.216	6	YES*
J	11.571	2	5.786	3.937	13.613	3	YES
K	1.739	2	0.870	0.592	2.046	8	NO
Error	5.878	4	1.470		6.915		
Total	85.000	26			100		

\*This parameter is considered as significant.

### 6. Artificial Neural Network modelling

In this study, the ANN was built and trained in Matlab® environment using Feed Forward Back Propagation (FFBP) neural network type [38,39]. A single hidden layer was selected. In order to find the optimal architecture, different numbers of neuron in the hidden layer were considered (the number of neuron in the hidden layer was varied from 8 to 20) and RMSE for each network was calculated. In this study, the ability of the learning program to predict output response, using different parameters viz learning rate ( $l_r$ ), momentum constant ( $m_c$ ) and epoch number, was also tested and optimized based on RMSE value (Equation 2). The ANN output with the lower RMSE value was identified as the best.

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (P - E)^2} \quad (2)$$

Where n is the total number of training case, P is the predicted value and E is the experimental value (i.e. FE simulation result of a training case (Table 1)).

The learning rate parameter plays an important role for the learning algorithm. This parameter controls how much to change the model in response to the estimated error each time the model weights are updated. Choosing the learning rate is challenging because, large value causes rapid convergence but the algorithm becomes unstable that may cause the increase of error and very small values can yield a more accuracy result, but longer time to converge [40]. The momentum constant is equally an important parameter that can affect learning algorithm. This parameter is used to prevent the system from converging to a local minimum or saddle point. A momentum coefficient that is too low cannot reliably avoid local minima, and can slow down the system training. As a result, to ensure a relatively stable and fast algorithm convergence, a gradient descent with momentum and adaptive learning rate back propagation 'traingdx' was applied. This function can i) train any network as long as its weight, net input, and transfer functions have derivative functions, and ii) It calculates derivatives of performance (RMSE) with respect to the weight and bias variable (X) [41]. Each variable is adjusted according to gradient descent with momentum (Equation 3).

$$dX = m_c \cdot dX_{pr} + l_r \cdot m_c \cdot \left( \frac{\partial RMSE}{\partial X} \right) \quad (3)$$

Where dXp presents the previous change in the weight or bias. In the described model, random weights are assigned to each processing element as an arbitrary starting point in the training process, and then progressively modified in light of several repetition of training cases data. For the transfer function, the Log-sigmoid 'logsig' function is used according to equation 4.

$$\log sig(x) = 1 / (1 + \exp^{-x}) \quad (4)$$

This transfer function, commonly used in multilayer networks that are trained using the back propagation algorithm, takes the input and squashes the output into the range 0 to 1.



The ANN architecture with all characteristic parameters is shown in Figure 5. A total data set of 27 gating system design parameter combinations was used to train the proposed ANN. Six further FE simulations with different design parameters dimension will be used later to validate the ANN process.

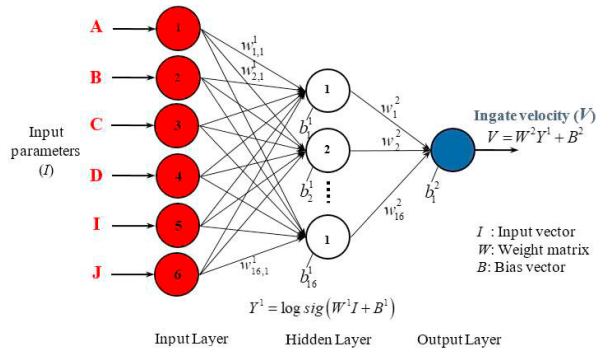


Fig. 5. The back-propagation ANN simulator used for the ingate velocity prediction

During the training process a constant adjustment of weightings between neurons lead to a reduction in RMSE. After training phase, the predicted ANN results are plotted versus the values of the FE training cases.

In this study, the optimized ANN architecture, with sixteen neurons in the hidden layer, has been tested and validated with six additional cases. The model is verified against randomly filling system design parameters simulated with FE code that means that the testing data file is completely independent from the train cases base. Figure 6 shows a plot of predicted ANN versus FE Ingate velocity for both training and testing (validation) data cases.

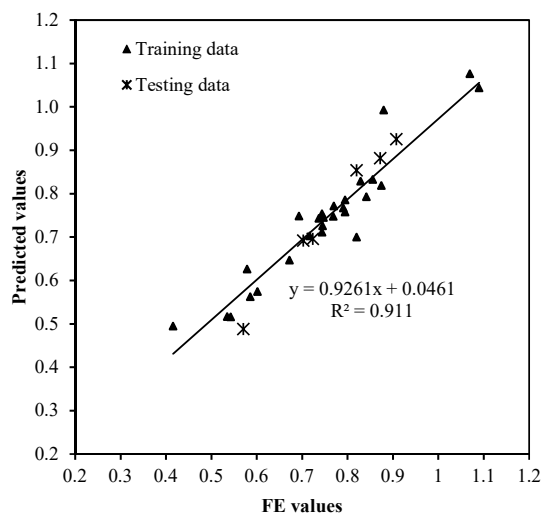


Fig. 6. FE vs. ANN predicted ingate velocity for train and test cases.

There is no overlap between the two sets of data and a reasonable predictions occur in both cases. The RMSE for the prediction of the entire case data sets is 4.04%. The value of the determination coefficient ( $R^2 = 0.91$ ) indicates that only 9% of the total variation are not explained by the optimized model.

In summary, it is clear that optimized ANN model can accurately predict the ingate velocity of a large number of gating system design parameter combinations. These predictions can be carried on quite rapidly (i.e. ANN model converge within a few seconds compared to approximately 4 hours for FE simulation) and easily compared to FE simulations. Finally, the found ANN model can greatly reduce the simulation time devote to reach the optimal design parameters combination in the entirely studied design space.

## 7. Conclusion

The optimal gating system design was usually determined, in traditional product development, by applying the casting rules followed by trial and error method to get accurate results. The most critical ingredient in the trial and error approach is how to employ previously returned errors to propose future trials. Therefore, more recent methods that can be used in design optimization such surface response methodology have appeared. These methods permit to intend the relationship between the input design variables and the response. This means that using the knowledge obtained by conducting a series of experiments the designer can optimize the response. However, it is impossible to carry out several experiences due to excessive cost of experimentation. As a result, FEM appears as a solution that permit to overcome this disadvantage. Nevertheless, by using this method, the optimal solution can be attained through one-by-one simulation that make the design development longer.

This is why, a relatively new methodology integrating FEM and ANN is applied, in the case of 3D sand casting of a simple plate with a molten aluminum alloy (EN AC-44200), in order to make the gating system design optimization faster than traditional methods. To attain this objective, FE flow simulations were performed using a combination of the gating system design parameters, selected as training cases from Taguchi OA. After that, the ANOVA tests were employed to select the significant design parameters on the ingate velocity. Finally, an ANN was trained and then validated by using the set of significant design parameters combination and their corresponding FE simulations. As a result, it is found that validated ANN simulator can rapidly predict an accurate ingate velocity, for all selected gating system design parameters combination covering the studied design space, which saves time and money for the designer.

In future work, this methodology will be applied in order to predict and optimize the non-conventional gating system design parameters made by 3D sand-printing techniques.

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